7.2.2 Percentages and Powers

Let's take an example for instance, the quantity of timber in a young forest grows almost exponentially. We may assume that the yearly rate of growth is 3.5 per cent. What increase is expected in ten years.

Consider (x) to be the initial quantity and (p) the percentage of increase. Then the increase factor per year is:

$$\frac{P}{100}\times X$$

and the total quantity in the first year is

$$x+\frac{P}{100}x=x\left(1+\frac{P}{100}\right)$$

To get from the initial quantity x to the increased quantity we have to multiply x by the factor $\left(1+\frac{P}{100}\right)$. For a second increase we have to multiply again with $\left(1+\frac{P}{100}\right)$. Thus we have

$$\left[x\left(1+\frac{P}{100}\right)\left(1+\frac{P}{100}\right)\right]=x\left(1+\frac{P}{100}\right)^2$$

The same operation may be applied over and over again. After n time intervals the equation may be written as:

$$x\left(1+\frac{P}{100}\right)^n \qquad (7.7)$$

Considering our problem if we consider the original amount of timber as 1 then after 10 years the amount increases to

$$1\left(1+\frac{3.5}{100}\right)^{10}=1.41$$

$$1.41-1=0.41=41\%$$

For any power equation a^n, a is called the base and n the exponent whereas the term power is reserved for the full expression:

a^n = power; a = base and n = exponent

Powers are useful to write large and small numbers in a convenient form such as: $1000 = 10^3$; $1/1000 = 10^{-3}$. The main advantage is that calculations are easier to perform with powers rather than with numbers consisting of too many decimals. In connection with powers we often mention the 'order of magnitude', thus two orders of magnitude would mean that the two numbers differ by a factor of $10^2 =$ 100. Powers can also be fractional for instance $8^{\frac{1}{3}} = 2$ this can also be written as $\sqrt[3]{8}$, so the roots and fractional powers are the same thing.

7.2.3 Significant Figures

Research workers are often concerned with the accuracy of figures. One source is errors in counting or uncertainties in the reading instrument. Another source is the random fluctuations in sampling. In practical applications we are usually forced to carry one digit beyond the last significant figure. For instance when a measurement or calculation gives a value of 3.47 mg with an error of almost ± 0.02 mg, the digit 7 is not significant, but dropping this figure and rounding off the quantity to 3.5 mg may cause a loss of relevant information. On the other hand, if the error were ± 0.1 mg, the digit 7 would be meaningless and should be dropped.

Rounding off is performed according to the following rules. The digits 1, 2, 3, 4 are rounded down that is the preceding figure is left unchanged. The digits 6, 7, 8, 9 are rounded up, that is the preceding figure is increased by 1. For the digit 5 some randomness is maintained. It can be enforced by the rule, 5 is rounded down whenever the preceding figure is even and is rounded up whenever the preceding figure is odd. Thus 4.65 and 4.75 are rounded off as 4.6 and 4.8 respectively.